

# MIXING BY TIDAL INTERACTION WITH SLOPING BOUNDARIES

Sonya A. Legg  
Woods Hole Oceanographic Institution  
MS 21

Woods Hole, MA 02543

phone: (508) 289-2801 fax: (508) 457-2181 email: [slegg@whoi.edu](mailto:slegg@whoi.edu)

Carl Wunsch  
Department of Earth, Atmospheric and Planetary Sciences  
Room 54-1524

Massachusetts Institute of Technology  
77 Massachusetts Ave.

Cambridge, MA 02139

phone: (617) 253-5937 fax: (617) 253-4464 email: [cwunsch@ocean.mit.edu](mailto:cwunsch@ocean.mit.edu)

Award #: N00149810096

<http://charybdis.whoi.edu/sonya/LIWI>

## LONG-TERM GOALS

The long-term goals of this project are to obtain an understanding of the mechanisms by which tidal energy is used to vertically mix the ocean against the action of gravity. Ultimately better parameterizations of the mixing caused by tides will result, allowing better prediction of coastal dynamics, biogeochemistry and sediment transport and the oceanic general circulation.

## OBJECTIVES

The process of mixing by tides interacting with topography involves several stages. First some fraction of the energy contained in the barotropic tide must be converted into baroclinic energy, through the generation of internal tides and turbulent boundary layers. Secondly, the energy in the internal tides must be transmitted into smaller vertical wavelengths, thereby increasing the vertical shear of the motion. When vertical shears are sufficiently strong, instability may result, leading to overturning and mixing. Finally, the mixed fluid is transported away from the mixing region modifying the ocean stratification. The net effect of the tides on the ocean stratification depends on the efficiency of all three processes.

Our objectives are to understand (a) the generation of internal tides by the interaction between barotropic tides and topography including finite-amplitude 3-dimensional variations in topography, finite-amplitude barotropic tidal forcing, non-hydrostatic effects and the boundary layer processes; (b) the mixing generated by internal tides reflecting from a sloping boundary in the presence of both 2 and 3-dimensional variations in slope, and finite rotation; and (c) the mechanisms of lateral and isopycnal transport of mixed fluid away from the boundary induced by the secondary circulations generated through spatial variations in mixing. Earlier studies have ignored 3-dimensional large amplitude variations in topography and non-hydrostatic effects (which are important for small-aspect ratio motion).

## **APPROACH**

We use high-resolution numerical simulations to explicitly resolve the turbulent mixing processes. For such simulations we require a numerical model which can (a) capture the non-hydrostatic physics of overturning and mixing processes (b) include arbitrary 3-dimensional variations in topography. The Marshall et al (1997a,b) code (known as the MIT ocean model), which is non-hydrostatic, and includes topography through a finite-volume formulation, is such a model.

We will carry out 3 different groups of simulations:

- (a) We will impose topography, barotropic tides and subinertial flows suggested by recent observations (e.g. Norfolk canyon region: Polzin et al (1998), and Monterey Canyon region, Petruncio et al, 1998), and investigate the internal tide generated by the flow-topography interactions, comparing these results with earlier models which assume small-amplitude (e.g. Bell, 1975) or 2-dimensional finite amplitude (Baines 1982) topography.
- (b) We will impose internal tide forcing and investigate the interaction with a 2-dimensionally varying slope, focusing on the influence of finite rotation on the overturning and mixing, and the effect of slope variations in localizing mixing, comparing with earlier laboratory and numerical studies with uniform slope and no rotation (Ivey and Nokes, 1989; Slinn and Riley, 1996).
- (c) We will impose internal tide forcing and investigate the interaction with 3-dimensional topography variations, focusing on the localization of mixing, and the resultant secondary circulation and lateral transports of mixed fluid.

## **WORK COMPLETED**

Much of the past year was spent in modifying the numerical code for the special geometries required here. Following that several nonhydrostatic simulations were completed, in both two and three dimensions, with finite amplitude topography and currents, including the following:

Nonhydrostatic calculations of the interaction of an internal wave with a critical slope in 2-dimensions, have been investigated to understand the influence of the curvature of the slope (e.g. planar, concave and convex). These calculations are now being repeated in 3-dimensions on Navy supercomputers.

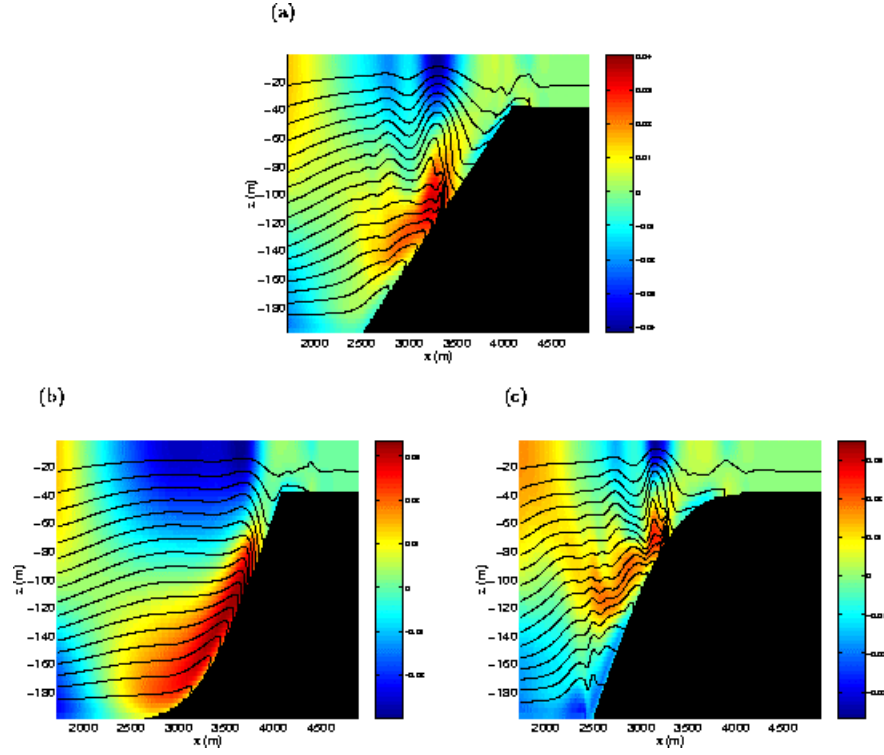
Three-dimensional investigations have begun to understand the influence of rotation on internal wave breaking on a planar slope, comparing mixing with strong rotation and without rotation.

Data from the LIWI funded TWIST project (Kurt Polzin, PI) have been used to initialize the stratification and topography for a region of the East Coast. We force the model with the barotropic tidal signal at the open boundary and investigate the internal wave response as the barotropic tidal signal encounters the continental shelf. The internal wave response to cross-shelf and along shelf tides has been examined, the former in 2D and the latter in 3D.

## **RESULTS**

### **1. Impact of variable topography on internal wave breaking.**

We have compared the internal wave reflection at a planar slope at the critical angle with that at a concave slope and a convex slope (where the average slope is also at the critical angle). In all three cases internal bore features with associated overturning develop (Figure 1), and a region of reduced stratification is generated just above the slope. However, for the concave slope, the bore is less



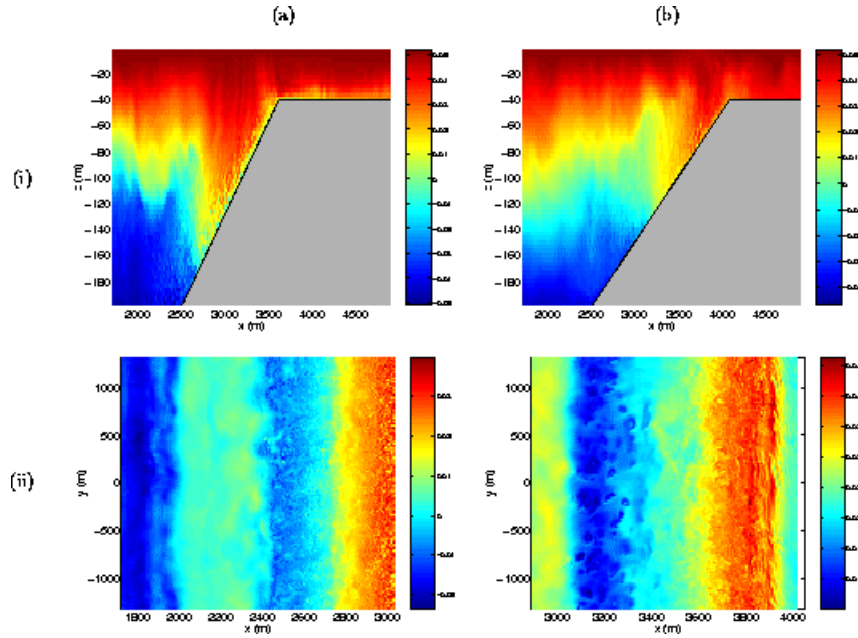
**Figure 1: Cross-slope velocity (m/s) (in color) and temperature (black contours) from 3 different simulations of an internal tide interacting with a slope. In all 3 simulations the stratification is  $N^2 = 10^{-6} \text{ s}^{-1}$ , the Coriolis parameter is  $f = 1 \times 10^{-4} \text{ s}^{-1}$ , the forcing frequency is  $\omega = 1.41 \times 10^{-4} \text{ s}^{-1}$  (the M2 tide), and the vertical wavelength of the forced internal wave is  $2 L_z$ , where  $L_z$  is the depth = 200m.**

**The average slope is at the critical angle. The simulations differ only in their topography: (a) planar slope, (b) concave slope, (c) convex slope. All three are shown at a time  $t = 3.14 T$ , where  $T$  is the tidal forcing period.**

pronounced, overturning is reduced, and in consequence modification of the stratification is less than the other two cases. This study was carried out for the gravest vertical mode internal wave, in which the temperature perturbations are strongest at mid-depths. The concave slope is critical near the bottom of the slope, where the wave temperature perturbations are small. The relative length scales of the slope variations and wave are therefore likely to be an important factor in determining whether curvature of the slope influences the mixing induced by a reflecting internal wave.

## 2. Impact of rotation on internal wave reflection and breaking.

Results have been compared from two three-dimensional calculations for internal waves reflecting from a planar slope at the critical angle. In the first case, the Coriolis parameter  $f = 1 \times 10^{-4} \text{ s}^{-1}$ , while in the second case,  $f = 0$ . The forcing frequency is the tidal frequency  $\omega = 1.41 \times 10^{-4} \text{ s}^{-1}$ . Although in both cases, an internal bore develops at the slope, the 3-dimensional structures associated with mixing differ (Figure 2). In the strongly rotating case, there is more organization of the structures into filaments, which are swept in the along-slope direction by the along-slope currents, while in the non-rotating case, the structures are of smaller scale, and more isotropic in character.



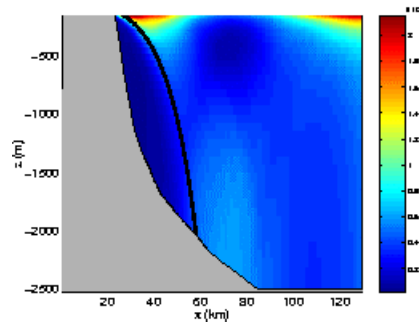
**Figure 2: Temperature from 3-dimensional simulations of an internal tide interacting with a planar slope, for Coriolis parameter (a)  $f = 0$ , (b)  $f = 1 \times 10^{-4} \text{ s}^{-1}$ , at a time  $t = 13.4 T$  where  $T$  is the tidal forcing period. In both cases the slope is at the critical angle. (i) Vertical ( $x,z$ ) section at  $y = -200\text{m}$ ; (ii) Horizontal ( $x,y$ ) section through the bore: (a (ii))  $z = -100\text{m}$ , (b (ii)),  $z = -50\text{m}$ .**

### 3. Internal tides in the TWIST region

The cross-shelf tidal forcing interacting with the 2-D average topography in the TWIST region generates internal tides primarily at the shelf-break, as predicted by Baines (1982). The stratification in this region is highly nonuniform, with much stronger stratification near the surface than at depth. The continental slope is concave, and is at the critical slope at mid-depths. However, the downward propagating wave energy emanating from the shelfbreak follows the convex shape of the wave characteristic, and hence does not encounter the critical region of the slope (Figure 3). Most of the internal wave energy is concentrated near the surface, where stratification is strongest. Hence, there is little benthic mixing generated by the cross-shelf barotropic tides interacting with 2-D slope. This implies that the observed bottom intensified mixing must result from the interactions with the fully 3-dimensional topography, a possibility we will investigate in our next series of simulations.

## IMPACT/APPLICATIONS

Our results should help the interpretations of observations of tidally forced flows on the continental slope observed by LIWI investigators and others. We anticipate our results will eventually allow better parameterizations of tidal mixing to be developed, allowing better prediction of coastal dynamics, biogeochemical processes and the oceanic general circulation (Munk and Wunsch, 1998).



**Figure 3:** *The kinetic energy density ( $\text{m}^2/\text{s}^2$ ) averaged over 4 tidal periods, generated by barotropic tidal forcing of amplitude  $U_{\text{max}} = 10\text{cm/s}$  at the eastern boundary,  $x = 250\text{km}$ . The 2-D topography is the along-slope averaged topography from the TWIST region, and the stratification is a polynomial fit to the actual stratification of that region. The solid black line shows the internal wave characteristic emanating from the shelf-break. Very little internal wave energy is found between this characteristic and the slope. The wave energy is concentrated in the upper 100m, where stratification is greatest, which are omitted to allow the deeper features to be visible.*

## TRANSITIONS

We are communicating our results to other members of LIWI, K. Polzin, L. Rosenfeld, and others, to help interpret their observations.

## RELATED PROJECTS

This project examines processes closely related to observations included in LIWI (Polzin, Toole and Schmitt and Paduan, Rosenfeld, Kunze and Gregg). The work is related to the NSF-supported general circulation studies of Wunsch.

## REFERENCES

- Baines, P. G., 1982. On internal tide generation models. *Deep Sea Res.*, 29, 307-338.
- Bell, T. H., 1975. Topographically generated internal waves in the open ocean. *J. Geophys. Res.*, 80, 320-327.
- Ivey, G. N., and R. I. Nokes, 1989. Vertical mixing due to the breaking of critical internal waves on sloping boundaries. *J. Fluid Mech.*, 204, 479-500.
- Marshall, J., A. Adcroft, C. Hill, L. Perelman and C. Heisey, 1997a. A finite-volume, incompressible Navier Stokes model for studies of the ocean on parallel computers. *J. Geophys. Res.*, 102, 5753.
- Marshall, J., C. Hill, L. Perelman and A. Adcroft, 1997b. Hydrostatic, quasi-hydrostatic and non-hydrostatic ocean modeling. *J. Geophys. Res.*, 102, 5733.

Munk, W., and C. Wunsch, 1998. Abyssal recipes II: Energetics of tidal and wind mixing. *Deep Sea Res.*, 45 1977-2010.

Petruncio, E. T., L. K. Rosenfeld and J. D. Paduan, 1998. Observations of the internal tide in Monterey Canyon. *J. Phys. Oceanogr.*, 28, 1873.

Polzin, K., J. Toole, R. Schmitt, E. Kunze and E. Montgomery, 1998. Turbulence and Waves over Irregularly Sloping Topography (TWIST), <http://hrp.whoi.edu/hrpgrp/liwi/twist1.html>.

Slinn, D. N., and J. J. Riley, 1996. Turbulent mixing in the oceanic boundary layer caused by wave reflection from sloping terrain. *Dyn. Atmos. Oceans*, 24, 51-62.